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Engineering Note

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Prediction of Center of Pressure for Deformed Solar Sails

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I. Introduction

THERE has been increasing interest in solar sailing due to its potential for propellantless space propulsion [1]. The thin membranes used for the solar sails have zero or very small bending rigidity and are buckled almost immediately under compression, which is a local deformation called wrinkling [2,3]. The membrane can also billow due to the solar pressure during operation or the gravity load during ground testing. In addition, it can deviate from the designed shape due to deployment error, thermal loads, or twisting of the boom supports. These deformations of the membrane structure could decrease the solar sail's performance in some cases, for example, inducing harmful roll moments [4]. The solar sail performance analysis considering the membrane deformation as outlined here is essential to the detail design of the solar sail system [5–7].

In the present Note, the authors propose a scheme to predict the solar sail performance, which includes the total thrust force, the center of pressure (CP), and the moment at the CP. We show in Sec. II that it is impossible to uniquely determine the CP (at which the residual moment becomes zero) for a deformed solar sail, because the direction and the magnitude of thrust forces are not constant for each point. Therefore, we present a scheme to decide the CP as being located where the total moment becomes minimum. According to the present scheme, the total thrust force, the CP, and the total moment are calculated in Sec. III for a deformed membrane, and the effects of miscalculating the CP are investigated.

II. Theory

A. Definition of Total Thrust Force and Total Moment at the Origin

The total thrust force $\mathbf{F}$ for a solar sail can be approximately calculated as the summation of the average thrust forces $\mathbf{f}_e$ acting on small finite elements of sail areas:

$$\mathbf{F} = \sum \mathbf{f}_e \tag{1}$$

where the summation is taken over the entire active sail area. The direction and the magnitude of the thrust forces $\mathbf{f}_e$ depend on the sail surface normal vector, the solar angle, the reflectivity, and so on, but those details are not given here.

The total moment $\mathbf{M}_{\mathrm{total}}$ at an arbitrary origin of coordinates is given as

$$\mathbf{M}_{\mathrm{total}} = \sum \mathbf{r}_e \times \mathbf{f}_e \tag{2}$$

where $\mathbf{r}_e$ is the position vector of the small elemental sail area, and $\times$ shows the outer product operator (see Fig. 1).

B. Total Moment at an Arbitrary Point

The total moment at an arbitrary point a located from the origin by a position vector $\mathbf{r}_a$ is

$$\begin{aligned}\mathbf{M}(\mathbf{r}_a) &= \sum(\mathbf{r}_e - \mathbf{r}_a) \times \mathbf{f}_e = \sum \mathbf{r}_e \times \mathbf{f}_e - \mathbf{r}_a \times \sum \mathbf{f}_e \\ &= \mathbf{M}_{\mathrm{total}} - \mathbf{r}_a \times \mathbf{F}\end{aligned} \tag{3}$$

If the point $\mathbf{r}_a$ could be chosen to satisfy the condition that $\mathbf{M}(\mathbf{r}_a) = \mathbf{0}$, the point a at $\mathbf{r}_a$ would be the CP. However, for a deformed sail, it is generally impossible to obtain a unique solution of the equation $\mathbf{M}(\mathbf{r}_a) = \mathbf{0}$; that is, it is impossible to uniquely define a CP.

C. Center of Pressure

One of the possible centers of pressure $\mathbf{r}_{\mathrm{min}}$ is defined by two conditions as follows:

Condition 1: The total moment $\mathbf{M}(\mathbf{r}_{\mathrm{min}}) = \mathbf{M}_{\mathrm{min}}$ at the center of pressure should be minimum (i.e., as near zero as possible).

Condition 2: The vector $\mathbf{r}_{\mathrm{min}}$ is perpendicular to the total force $\mathbf{F}$.

From Condition 1, three equations are derived, as shown next:

$$\frac{\partial}{\partial r_x} |\mathbf{M}(\mathbf{r}_{\mathrm{min}})|^2 = 0 \tag{4a}$$

$$\frac{\partial}{\partial r_y} |\mathbf{M}(\mathbf{r}_{\mathrm{min}})|^2 = 0 \tag{4b}$$

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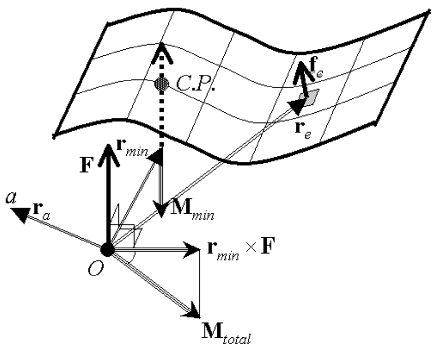

Fig. 1 Definition sketch for force and moment components on a deformed solar sail membrane.

$$\frac{\partial}{\partial r_z}|\mathbf{M}(\mathbf{r}_{\min})|^2 = 0 \tag{4c}$$

Substitution of Eq. (3) for Eq. (4a) gives

$$\begin{gathered}
\frac{\partial}{\partial r_x}\{(\mathbf{M}_{\text{total}} - \mathbf{r}_{\min} \times \mathbf{F}) \bullet (\mathbf{M}_{\text{total}} - \mathbf{r}_{\min} \times \mathbf{F})\} = 0 \\
2\left(\frac{\partial \mathbf{r}_{\min}}{\partial r_x} \times \mathbf{F}\right) \bullet (\mathbf{r}_{\min} \times \mathbf{F} - \mathbf{M}_{\text{total}}) = 0 \\
2(\mathbf{e}_x \times \mathbf{F}) \bullet (\mathbf{r}_{\min} \times \mathbf{F} - \mathbf{M}_{\text{total}}) = 0 \qquad (\mathbf{e}_x = [1 \quad 0 \quad 0]^T) \\
2\mathbf{e}_x \bullet (\mathbf{F} \times (\mathbf{r}_{\min} \times \mathbf{F} - \mathbf{M}_{\text{total}})) = 0 \\
(\because (\mathbf{A} \times \mathbf{B}) \bullet \mathbf{C} = \mathbf{A} \bullet (\mathbf{B} \times \mathbf{C})) \\
2[1 \quad 0 \quad 0] \bullet (\mathbf{F} \times (\mathbf{r}_{\min} \times \mathbf{F} - \mathbf{M}_{\text{total}})) = 0 \\
(\because \mathbf{A} \bullet \mathbf{B} = \mathbf{A}^T \bullet \mathbf{B}) \qquad \text{(4a\&x2032)}
\end{gathered}$$

where $\bullet$ denotes the inner product operator.

By the same procedure, the following two equations can be derived from Eqs. (4b) and (4c):

$$2[0 \quad 1 \quad 0] \bullet (\mathbf{F} \times (\mathbf{r}_{\min} \times \mathbf{F} - \mathbf{M}_{\text{total}})) = 0 \qquad \text{(4b\&x2032)}$$

$$2[0 \quad 0 \quad 1] \bullet (\mathbf{F} \times (\mathbf{r}_{\min} \times \mathbf{F} - \mathbf{M}_{\text{total}})) = 0 \qquad \text{(4c\&x2032)}$$

Combining Eqs. 4(a′)–4(c′) in matrix form,

$$\begin{gathered}
2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet (\mathbf{F} \times (\mathbf{r}_{\min} \times \mathbf{F} - \mathbf{M}_{\text{total}})) = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \\
\therefore \mathbf{F} \times (\mathbf{r}_{\min} \times \mathbf{F} - \mathbf{M}_{\text{total}}) = \mathbf{0}
\end{gathered} \tag{5}$$

By the distributive law of the triple outer product

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \bullet \mathbf{C})\mathbf{B} - (\mathbf{A} \bullet \mathbf{B})\mathbf{C}$$

Eq. (5) can be modified as

$$(\mathbf{F} \bullet \mathbf{F})\mathbf{r}_{\min} - (\mathbf{F} \bullet \mathbf{r}_{\min})\mathbf{F} - \mathbf{F} \times \mathbf{M}_{\text{total}} = \mathbf{0} \tag{5′}$$

It is impossible to get the unique solution $\mathbf{r}_{\min}$ for Eq. 5(d′). Considering Condition 2, it has to be that $\mathbf{F} \bullet \mathbf{r}_{\min} = 0$, then the second term of Eq. 5(d′) vanishes, leaving

$$(\mathbf{F} \bullet \mathbf{F})\mathbf{r}_{\min} - \mathbf{F} \times \mathbf{M}_{\text{total}} = \mathbf{0}$$

Therefore,

$$\mathbf{r}_{\min} = \frac{\mathbf{F} \times \mathbf{M}_{\text{total}}}{|\mathbf{F}|^2} \tag{6}$$

Every point on the line that is through the position $\mathbf{r}_{\min}$ given by Eq. (6) and that is parallel with the total thrust force vector $\mathbf{F}$ gives the minimum total moment. Finally, the CP is defined as follows:

Definition 1: The CP is the intersection between the undeformed solar sail surface and the line defined by the position $\mathbf{r}_{\min}$ and the direction of the total thrust force vector $\mathbf{F}$.

Substituting Eq. (6) into Eq. (3), the minimum moment, which is the moment at the CP, is given as

$$\begin{aligned}
\mathbf{M}_{\min} &= \mathbf{M}_{\text{total}} - \frac{(\mathbf{F} \times \mathbf{M}_{\text{total}}) \times \mathbf{F}}{|\mathbf{F}|^2} \\
&= \mathbf{M}_{\text{total}} - \frac{(\mathbf{F} \bullet \mathbf{F})\mathbf{M}_{\text{total}} - (\mathbf{F} \bullet \mathbf{M}_{\text{total}})\mathbf{F}}{|\mathbf{F}|^2} = \frac{\mathbf{F} \bullet \mathbf{M}_{\text{total}}}{|\mathbf{F}|^2}\mathbf{F}
\end{aligned} \tag{7}$$

Considering Eq. (7), we make the following observations:

1) The moment at the CP is parallel with the total thrust force $\mathbf{F}$.

2) Only if the total thrust force $\mathbf{F}$ and the total moment at the origin $\mathbf{M}_{\text{total}}$ are orthogonal to each other will the moment at the CP be zero.

III. Solar Sail Performance of Twisted Square Membrane

Solar sail performance descriptors such as the total thrust force, the position of the CP, and the moment at the CP are investigated for a twisted square membrane of side length L_0, as shown in Fig. 2. This twisted membrane could represent a deployment error or severe twisting of the solar sail boom. Consider that the membrane is twisted in such a way that the Z-direction deformation is given as

$$W = W_0\left(\frac{X}{L_0}\right)\left(\frac{Y}{L_0}\right) \tag{8}$$

where W_0 is the maximum Z deformation. In this study, we assumed $W_0 = 2$ m and the half-length of the square membrane side $L_0 = 5$ m. The sun angle θ_x is varied from 0 to 60 deg around the X axis. The solar incident flux was specified as 1400 W/m^2, and it is assumed that the membrane has perfect reflectivity. The total thrust force $\mathbf{F}$ and the total moment at the origin $\mathbf{M}_{\text{total}}$ can be calculated using the integrated optical design analysis (IODA-SAIL) software developed by SRS Technologies [8].

Figure 3 shows the variation of the three direction components of the thrust force for various sun angles. As the sun angle increases, the thrust force in the Z direction decreases. On the other hand, an offset sun angle generates a Y-direction force. The X-direction force is constantly null.

Figure 4 shows the position of the CP calculated with the proposed method using the IODA-SAIL software for various sun angles. As the sun angle increases, the center of pressure moves from the center

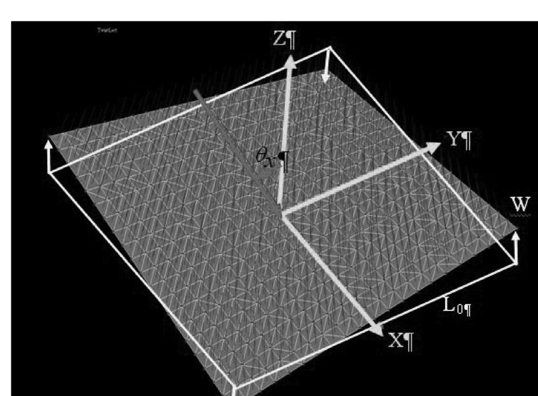

Fig. 2 Definition sketch for twisted square membrane. The light frame denotes the boundary of the original undeformed membrane.

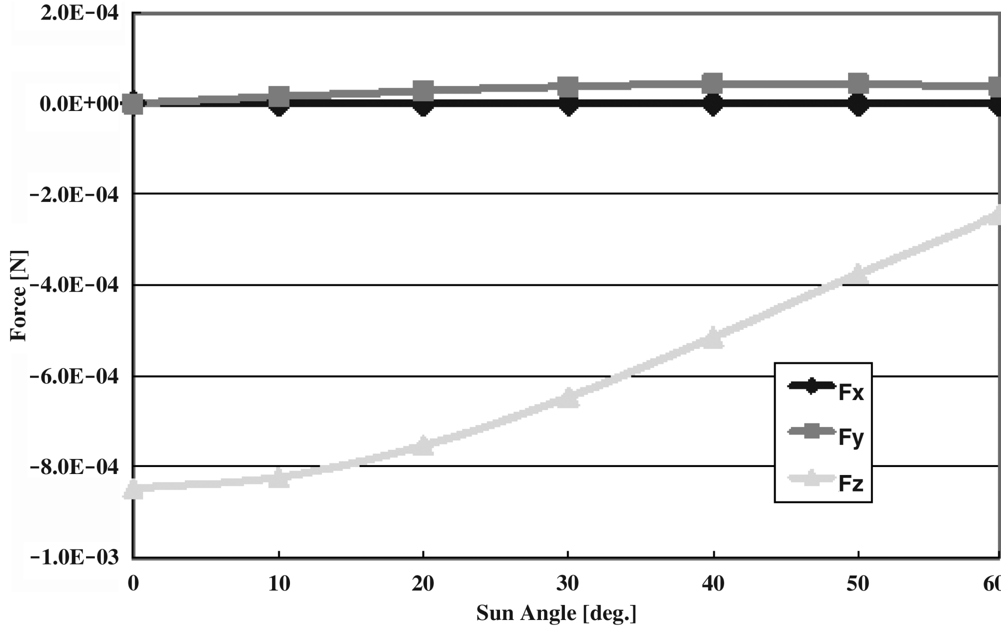

Fig. 3 Three orthogonal components of the total thrust force as a function of solar angle.

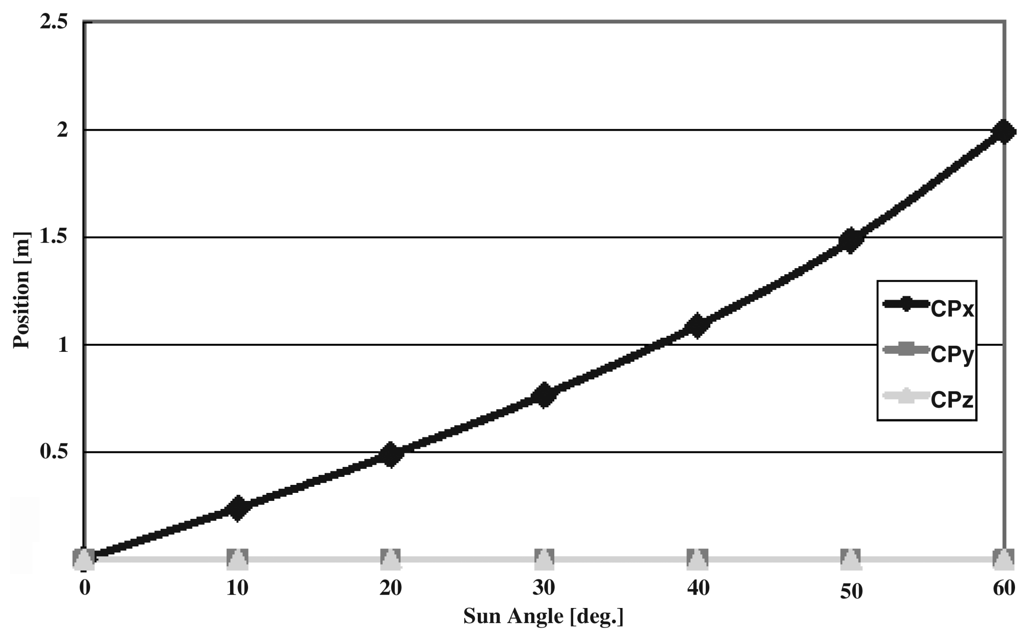

Fig. 4 Position of the center of pressure as a function of solar angle ($CP_x = X_{CP}$, $CP_y = Y_{CP}$, $CP_z = Z_{CP}$).

of the membrane in the *X* direction. This movement is caused by the asymmetry of the sun angle on the membrane surface. Figure 5 shows the moment at the center of pressure. The offset sun angle induces the rolling moment at the CP. The direction of the rolling moment coincides with the direction of thrust force and its magnitude increases as the sun angle increases.

Figures 6 and 7 show the error when the moment is calculated about a position other than that defined by Definition 1. Such errors could have significant consequences for guidance, navigation, and control schemes.

IV. Conclusions

Solar sail performance analysis considering the sail deformation is essential to the detail design, analysis, and ultimate success of the solar sail system. In the present Note, we proposed a scheme to predict the solar sail performance, which includes the total thrust force, the center of pressure, and the total moment at the CP. We showed that it is impossible to uniquely determine the CP (at which the residual moment becomes zero) for a deformed solar sail, because the direction and the magnitude of the thrust forces are not constant for each point on the sail surface. Therefore, we presented a scheme to decide the CP location at which the total moment becomes minimum (i.e., as near zero as possible). According to the present scheme, the total thrust force, the CP, and the total moment were calculated for a deformed membrane, and the effects on performance predictions from locating the CP elsewhere were investigated. It was shown that the method presented can predict the effect of the solar sail performance by deformations such as asymmetric billow, deployment error, and the sun angle. The movement of the center of pressure and the roll moment at the CP can present a large effect to the

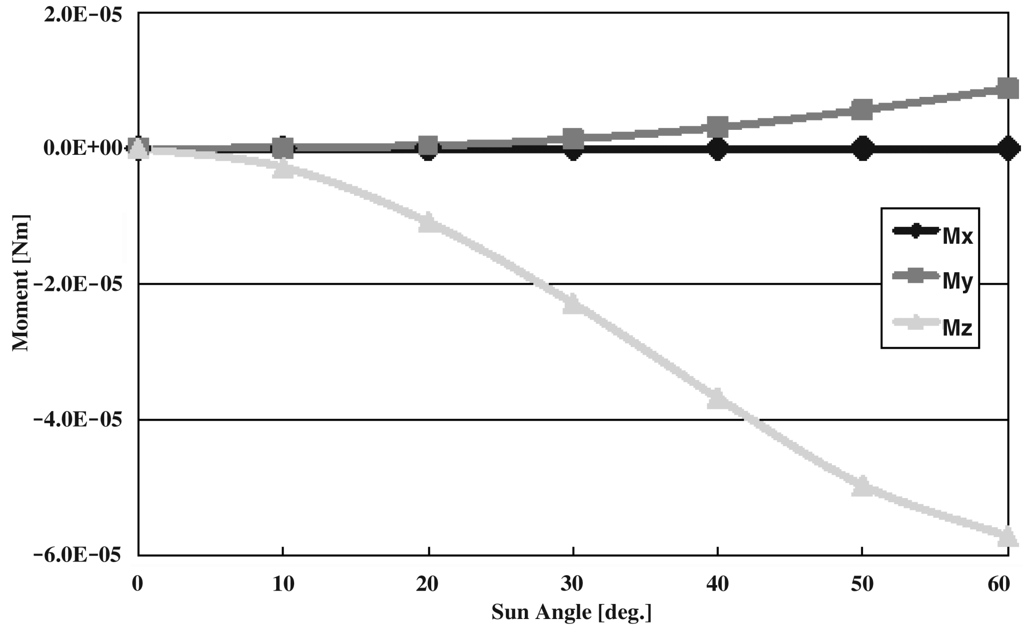

Fig. 5 Three orthogonal components of the moment at the CP as a function of solar angle.

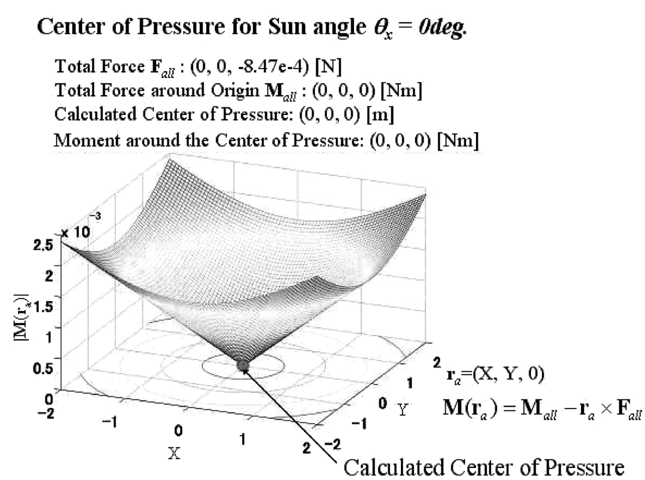

Fig. 6 Error in moment calculated at other than the CP defined by Definition 1, for normal sun incidence.

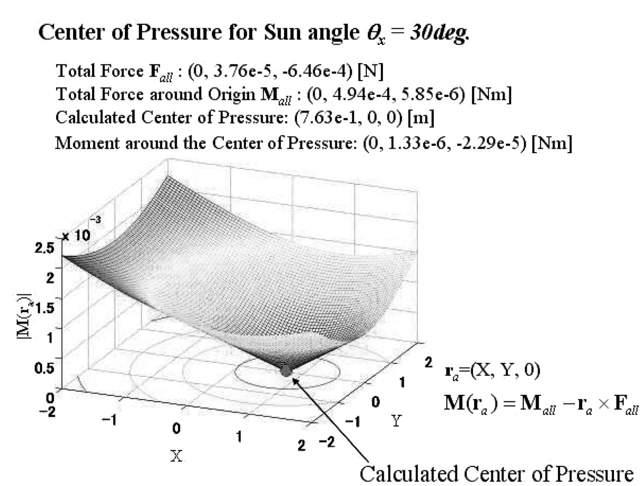

Fig. 7 Error in moment calculated at other than the CP defined by Definition 1, for off-normal sun incidence.

attitude control of the solar sail. Discrepancies in the location of the CP can lead to large errors in the predicted moments.

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